Course title: Constrained Optimization and Linear Algebra					
Course code: MPE 111		**No. of credits:** 4	**L-T-P:** 42-14-0	**Learning hours:** 56	
Pre-requisite course code and title (if any): None. Knowledge of Mathematics at the level of 10+2 is required.					
Department: Department of Policy Studies					
Course coordinator: Dr.Soumendu Sarkar		**Course instructor**: Dr.Soumendu Sarkar			
Contact details: soumendu.sarkar@terisas.ac.in					
Course type: Core		**Course offered in:** Semester 1			

Course description:
The use of optimization techniques in economics can be motivated by Robbins' (1932) definition of economics as "the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses". This course brings together central results in Linear Algebra and Real Analysis to provide the foundation of constrained optimization techniques used in modern economics. However, Linear Algebra and Real Analysis are important topics in their own right, and many results thereof are used in different branches of economics. Besides equipping the student with economists' essential toolbox, this course emphasises on understanding important mathematical properties that motivate the underlying assumptions of economic models.

Course objectives:
1. Understanding major concepts of Linear Algebra and Real Analysis.
2. To appreciate the criticality of the role of mathematical assumptions in economic modelling.
3. To provide foundations of major techniques to solve optimization problems in economics.
4. To familiarise students with logical arguments and proofs.

Course contents

Module	Topic	L	T	P
	Group 1			
I	**Preliminaries** (a) Symbolic logic; (b) Necessary vs. sufficient conditions; (c) Methods of proof	2		
	Group 2			
II	**Linear Algebra** (a) Vectors; Vector Spaces; Linear Dependence; Rank and Basis; Inner Product and Norm. (b) Matrices; Basic operations; Rank of a matrix; Inverse of a matrix. (c) Systems of Linear Equations; Existence, uniqueness and calculation of solutions; Determinants; Matrix Inversion; Cramer's Rule. (d) Eigenvalues and Eigenvectors; Relationship with Trace and Determinant; Symmetric matrices; Spectral Decomposition; Quadratic Forms and their Definiteness	8	3	
	Group 3			
III	**Real Analysis** (a) Real Space; (b)Sequence and Limit; Sequence and Limit in Vector Space; (c) Open Set; Closed Set; Compact Set in Vector Space; Bolzano-Weierstrass Theorem; (d) Continuous functions; Weierstrass' Theorem.	6	3	
IV	**Differential Calculus** (a) Single variable case: Slope of a function and its derivative; Continuity and Differentiability; approximation by differential; higher order derivatives. (b) Multiple variables case: Partials; Total Derivative; higher order derivatives. (c)Vector-valued functions; Jacobian Matrix. (d) Composite functions; Chain Rule. Inverse function and its derivative. (e) Implicit function; Implicit functions of several variables; Systems of Implicit Functions; Solutions of Systems of Implicit Functions: the Implicit Function Theorem.	8	3	
V	**Convex Analysis**	4	1	

	Convex Sets; Intermediate Value Theorem; Mean Value Theorem; Taylor's Expansion. Concave functions; Concave functions on convex sets; differentiable functions on convex sets and concavity. Quasi-concave functions on convex sets; differentiable functions on convex sets and quasi-concavity.			
	Group 4			
VI	**Unconstrained Optimization** (a)Local and Global maximum; Existence and uniqueness; (b)Necessary and sufficient conditions for local maximum; (c) Necessary and sufficient conditions for global maximum	2	1	
VII	**Constrained Optimization** (a) Optimization with equality constraints; Necessary and sufficient conditions for constrained local maximum; sufficient conditions for constrained global maximum. (b) Optimization with inequality constraints; saddle point; constrained global maximum and saddle points; Kuhn-Tucker Conditions and Saddle Points; Sufficient conditions for constrained global maximum; Necessary and sufficient conditions for constrained local maximum.	8	3	
VIII	**Additional Topics** (a) Integration; introduction to differential equations (b) Linear Programming (c) Optimal Control and Dynamic Programming Problems	4		
	Total	42	14	

Evaluation criteria:
Test 1: Homework Assignments: 30%
Test 2:Written Examination [Group 2] 20%
Test 3: Written Examination [Group 3] 30%
Test 4: Written Examination [Group 4] 20%.

Learning outcomes:
At the end of this course, students will be able to
1. Master the essential concepts and techniques of Linear Algebra, Real Analysis and Optimization and apply them to important economic problems [Tests 1-4]
2. Understand and appreciate the motivation of essential mathematical assumptions made in economic modelling [Test 4]

Pedagogical approach:
Classroom teaching; solving problem sets; classroom interaction and quizzes.

Materials:
Texts:
1. Simon, C.P. and Blume, L., 1994. *Mathematics for economists*, New York: Norton.

Additional information (if any):
Lecture notes and problem sets will be provided.

Student responsibilities: Attendance, feedback, discipline: as per university rules.

Course reviewers:
1. Tridip Ray, Professor, Economics and Planning Unit, Indian Statistical Institute, New Delhi
2. Subrata Guha, Associate Professor, Centre for Economic Studies and Planning, Jawaharlal Nehru University, New Delhi